Al-Jabar: Jurnal Pendidikan Matematika
Volume 13, Number 1, 2022, Pages 23 – 31
http://ejournal.radenintan.ac.id/index.php/al-jabar/index

Learning ordinary differential equation at undergraduate level: A systematic learning review

Elika Kurniadi[1]*, Zulkardi[1], Ratu Ilma Indra Putri[1]
[1] Universitas Sriwijaya, Indonesia
✉ elikakurniadi@fkip.unsri.ac.id*

Artikel Information
Submitted December 18, 2021
Revised Febbruary 27, 2022
Accepted March 21, 2022

Keywords
Articles Scopus Q1;
Ordinary Differential Equation;
Systematic Literature Review.

Abstract
The research about Ordinary Differential Equation (ODE) has increased more widely since 1970. Therefore, several published articles in some journals can be found in some sources. This paper aims to provide an overview of the learning differential equation in higher education based on relevant literature. Therefore, we are interested in conducting a Systematic Literature Review (SLR) methodology from 24 articles generated from Top 5 Scopus Q1 according to SJR reported by Scimago Journal & Country Rank in the subject area of education. The present study focuses on two aspects, namely: 1) the learning method of ODE that is proposed in the academic literature, and 2) the topic of ODE has been put forward and discussed in the academic literature. The systematic literature review found four methods of learning ODE (active learning, mathematical modeling, information, and technology communication, and geometric and qualitative solutions). Moreover, we also concluded that the several topics of ODE in the academic literature are 1) first order of ODE, 2) Euler method, 3) application to the problem (rate of change, population model, logistic generalized, and spring-mass), 4) second-order of ODE, and 5) system of ODE. The result of this study can provide a summary of existing literature and identify the weakness or gap to be investigated further in the following research related to the topic ODE.

INTRODUCTION

Ordinary Differential Equation (ODE) is an equation that consists of one or more functions of a single independent variable along with their derivatives. It is an essential topic in mathematics related to different areas of science such as physics, biology, chemistry, engineering, economics, and others (Habre, 2000; Rasmussen & Keene, 2019; Rasmussen, 2001). Nowadays, the change of learning ODE has mainly been influenced by the active learning didactic process and using technology as a tool for teaching and learning. The development of technology, for instance, applications in numerical methods that are valuable tools for solving the differential equation and understanding some properties in differential equations (Hoyles, 2018; Quinn & Aarão, 2020; Soares & Borba, 2014). Moreover, the innovation of technology is enriched the learning environment in the classroom.

The separation of technology and teaching is a drain on education and student skills. Technological change over time prompted teachers to try many teaching approaches. The most important techniques are to engage students with the subject and improve their understanding and reasoning. Some studies proposed various frameworks for integrating computers and technology into their classrooms (Crouch & Mazur, 2001; Kashefi et al., 2012). By linking technology to learning differential equations, the result indeed points to graphics software, be it for computers or calculators. However, integrating these well-known technological resources should not be taken as a guarantee of a better learning process. Innovation can only occur if the

How to cite Kurniadi E., Zulkardi Z. & Putri R. I. I. (2022). Learning ordinary differential equation at undergraduate level: A systematic learning review. *Al-Jabar: Jurnal Pendidikan Matematika*, *13*(1), 23-31.
E-ISSN 2540-7562
Published by Mathematics Education Department, UIN Raden Intan Lampung

software is carefully and adequately implemented in the curriculum and brings a significant and noticeable improvement in the teaching and learning process.

In the last decades, the transformation of teaching and learning differential equations at the undergraduate level happened actively with different subjects and perspectives from traditional teaching, and learning differential equations is undertaken by numerous researchers worldwide. Many studies are conducted to overcome some research questions related to the exploration of the new learning methods or the relationship between the technology and teaching in the differential equation or the trend and challenge of teaching ODE (Ju & Kwon, 2007; Rasmussen & Kwon, 2007; Stephan & Rasmussen, 2002). However, there are no studies that show evidence regarding the teaching of ODE in universities. This study aims to present evidence related to teaching ODE in higher education, such as design learning, student responses to various pedagogical methods in teaching and learning ODE, the results of the effectiveness of innovation, and the main focus on mathematics topics in ODE. To gain the best knowledge from the result of this research, we should conduct a literature review (Xiao & Watson, 2019). Meanwhile, there is less literature review in those studies. In other words, a literature review to identify, discover, critically assess, and synthesize the relevant studies topics regarding teaching and learning of ODE remains open.

One of the increasing trend research since around 1990 is the review of the literature (Khan et al., 2003). The present study analyzes existing literature related to teaching and learning ODE in higher education from the 1970s to 2020. Therefore, this research aims to provide an overview of learning differential equations in higher education, especially in the method of learning and topic of ODE based on relevant literature. In principle, a systematic review is a research method that summarizes primary research results to present more comprehensive facts and is balanced systematically. A Systematic Literature Review (SLR) was chosen as a suitable methodology which has five steps (Khan et al., 2003), namely: 1) framing questions for review, 2) identifying relevant work, 3) assessing the quality of studies, 4) summarizing the evidence, and 5) interpreting the findings.

METHODS

The systematic literature review aims to provide an overview of learning differential equations in higher education based on relevant literature. According to this purpose, the following step of an SLR is conducted (Khan et al, 2003).

1. Framing question for review

The questions of this systematic literature review are that:

- What method of learning ODE is proposed in the academic literature?
- What topics of ODE have been discussed in the academic literature?

2. Identifying relevant work

We have selected the Scopus database to determine the relevant literature (accessed on 25th November 2021). We follow this search flowchart and select an article to record articles for a systematic literature review. A guide seeks procedure become used to discover articles which addressed Ordinary Differential Equation (ODE) or learning undergraduate level with inside the title. Three impartial coders study the total-textual content via the abstract, or if needed, to categorize every article. This helped to calibrate the supply choice procedure and to peer if the

selected articles have been certainly approximately ODE. The inclusion standards have been the following (1) specializing in defining ODE and its aspects; (2) published in English language; (3) Published in an academic journal.

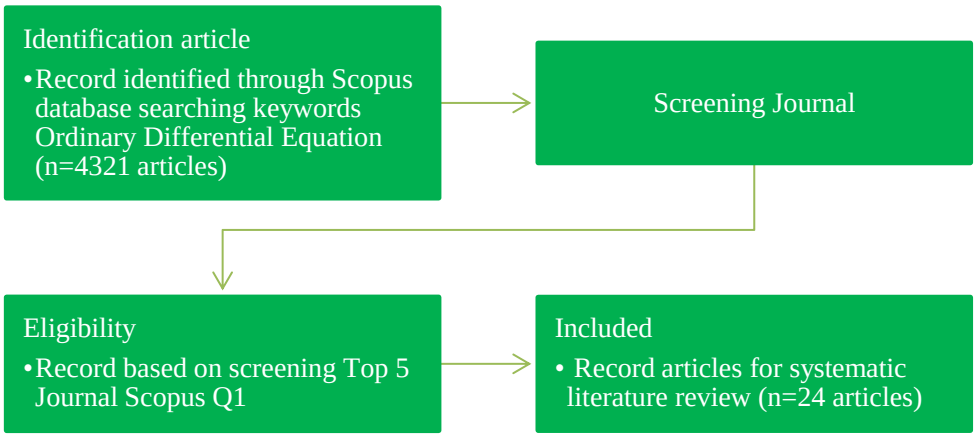

Figure 1. Flowchart relevant article

3. Assessing the quality of studies

The quality of the reviewed literature is essential for a Systematic Literature Review (SLR) since the quality of the conclusions is related to the quality of the selected literature. Therefore to assess the quality of studies, we considered several aspects of the journal and author. We prefer to choose a journal from the top 5 journals Scopus Q1 in the subject of the area in mathematics education. Here is the profile of the journals and the title of 24 articles that are reviewed.

Table 1. Top 5 Journal Scopus Q1 According to the SJR reported by Scimago Journal & Country Rank in the Subject Area of Education

Rank	Journal	SJR 2020	Publisher
1	Journal for research in mathematics education	2.69	National Council of Teachers of Mathematics
2	Educational studies in mathematics	1.85	Springer Netherlands
3	Journal of mathematics teacher education	1.72	Springer Netherlands
4	Journal of Mathematical Behavior	1.33	Elsevier Inc.
5	ZDM–Mathematics Education	1.2	Springer Verlag

Table 2. The Title of Article from Top 5 Journal Scopus Q1 and Number of Citation in Google Scholar (Accessed on 15th December 2021)

Rank	Journal	Code	Title of Article	Citation
1	Journal for research in mathematics education (n = 3 articles)	A1	Pedagogical content tools: Integrating student reasoning and mathematics in instruction.	202
		A2	A teacher needs the knowledge to provide analytic scaffolding during undergraduate mathematics classroom discussions.	187
		A3	Transforming the mathematical practices of learners and teachers through digital technology	53
2	Educational studies in mathematics (n = 3 articles)	B1	A Beyond motivation: Exploring mathematical modeling as a context for deepening students' understandings of curricular mathematics.	260

Rank	Journal	Code	Title of Article	Citation
		B2	Algebraic manipulation as motion within a landscape	34
		B3	A comparison of a classical calculus test with a similar multiple-choice test.	7
3	Journal of mathematics teacher Education (n = 1 article)	C1	Critical stance within a community of inquiry in an advanced mathematics course for pre-service teachers.	2
4	Journal of Mathematical Behavior (n = 12 articles)	D1	How can emphasizing mathematical modeling Do principles benefit students in a traditionally taught differential equations course?	32
		D2	Representations of a mathematical model as a means of analyzing growth phenomena.	7
		D3	Knowing solutions to differential equations with the rate of change as a function: Waypoints in the journey.	4
		D4	New directions in differential equations: A framework for interpreting students' understandings and difficulties.	214
		D5	Exploring students' strategies to solve ordinary differential equations in a reformed setting.	94
		D6	Ways of talking and ways of positioning: Students' beliefs in an inquiry-oriented differential equations class	46
		D7	Characterization of dynamic reasoning: Reasoning with time as a parameter	32
		D8	The function of graphs and gestures in algorithmatization.	22
		D9	Classroom mathematical practices and gesturing.	117
		D10	Reinventing solutions to systems of linear differential equations: A case of emergent models involving analytic expressions.	64
		D11	An inquiry-oriented approach to undergraduate mathematics.	216
		D12	Classroom mathematical practices in differential equations.	191
5	ZDM–Mathematics Education (n = 5 articles)	E1	The role of software Modellus in a teaching approach based on model analysis.	13
		E2	Teaching mathematical modelling through project work.	196
		E3	Considerations on the use of mathematics in modeling activities.	25
		E4	Making connections among representations of eigenvector: What sort of a beast is it?	7
		E5	Blended learning in the first-year engineering mathematics.	10

4. Summarizing the evidence

We have already selected and gathered the relevant work with our systematic literature review, as we can see the title of articles from Table 2. The articles consist of the results of the research.

In this systematic literature review, we summarize the evidence based on the questions in step 1 related to the method of learning, the topic, and the important findings in teaching and learning ODE.

5. Interpreting the findings

The consideration of the review question leads us to describe the corresponding finding, think critically, and look within findings or across finding. In thinking about interpreting the finding, we analyzed the summary of evidence deeply. Interpretation needs more integrative thinking and is more conceptual than data analysis because it builds argument from the researcher's understanding and insight.

RESULTS AND DISCUSSION

Based on the data of the year published articles reviewed in this paper (n = 24 articles), we can generate the trend of published articles in Scopus Q1 related to ODE learning below.

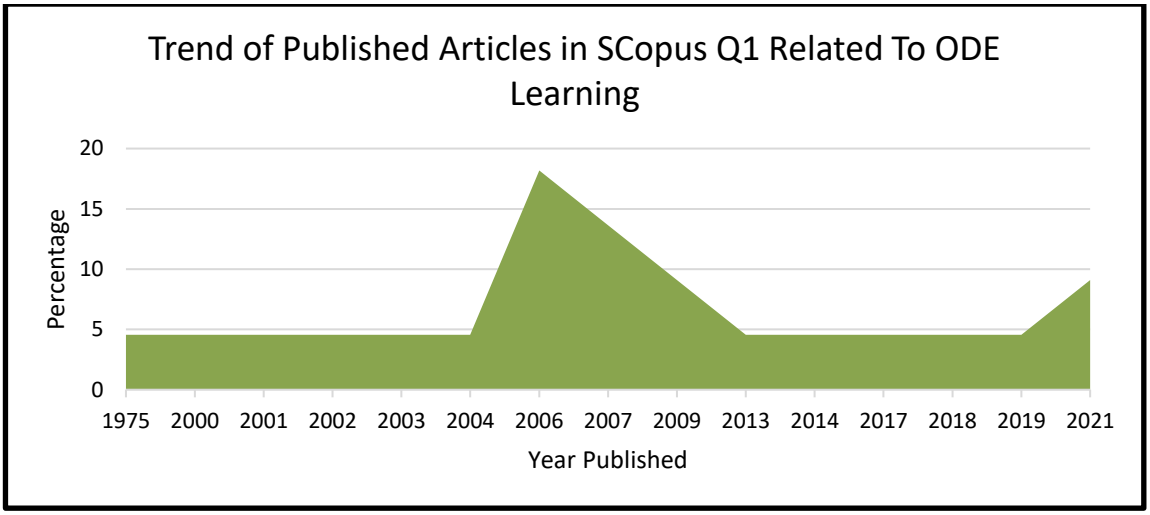

Figure 2. Graphic of year published article

Since 1975, a published article discussed the study of literature mathematical education, especially the part of a different method of testing, including evaluation of ODE learning. According to the graphic above, we can find out that most article about ODE learning has been published from 2004 to 2009. The trend of published articles in Scopus Q1 related to ODE learning will increase as we can see the point of percentage in 2021. More detail about the author in reviewed articles, Chris Rasmussen from San Diego State University is recorded as the author with the highest number of articles (28%) from this systematic literature review. Moreover, we present the information regarding the location of origin from the authors' affiliation as the below pie chart.

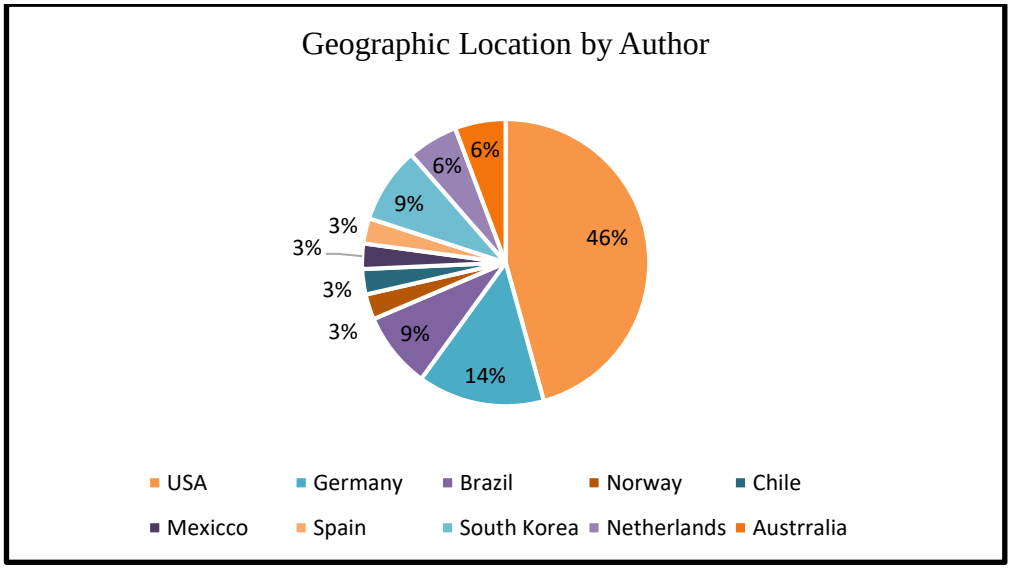

Figure 3. Pie Chart of Geographic Location by Author

According to the pie chart above, we can find that the USA (46%), Germany (14%), South Korea (9%), and Brazil (9%) are the regions with the highest number of records regarding the geographic location by author. Netherlands and Australia have the same number of records from the original location from the author's affiliation in the article reviewed. Furthermore, the following four regions appear with 3%; Norway, Chile, Mexico, and Spain.

In this section, we will attempt to elaborate the answer to the question for review that is: 1) What is the method of learning ODE that is proposed in the academic literature, and 2) What topics of ODE have been put forward and discussed in the academic literature.

1. Method of learning ODE

Table 3. Method of learning ODE

No	Method of learning	Paper Code	Number of records
1.	Active learning	A1, A2, B1, C1, D4, D6, D7, D9, D10, D12, E4	11
2.	Mathematical Modeling	B3, D1, D2, E3	4
3	Information and Technology Communication	A3, B2, D2, ,E1, E2, E5	6
4	Geometric and qualitative solutions	D3, D5, D8, D11	4

The method of learning will influence the learning environment in the classroom. Based on table 3, we can classify four methods of learning ODE. First, active learning is the greatest number of records in the review literature; some active learning proposed are inquiry-based learning, problem-based learning, collaborative learning, problem-solving, etc. The active learnings ask students to practice mathematical activities. Meanwhile, they discover mathematics concepts (Ju & Kwon, 2007). The authors report several advantages from active learning;1) the students gain a notable cognitive in thinking and understanding, 2) the teacher guiding discussion help students to reinvent knowledge, and 3) improve the concept of ODE (Goodchild et al., 2021; Rasmussen & Kwon, 2007; Stephan & Rasmussen, 2002). Second, mathematical modelling is a method of learning that involve students to solve the real authentic problem by implementing mathematical knowledge related to ODE to develop modeling competencies, critical understanding in a

different situation, and motivation through the teaching and learning process (Czocher, 2017; de Almeida, 2018; Zbiek & Conner, 2006). Third, the use of digital tools in learning ODE is to obtain the analytical and graphical solution of ODE (Quinn & Aarão, 2020). Several simple digital tools can explore ODE and their solution, such as Excel, Wolfram- Alpha, and Geogebra. In da Silva Soares & Borba's study, it focuses on the Modellus Software to think visualization provided by the software as we know that the visualization of ODE is an important thing to grasp the concept of the differential equation. Therefore, the last method of learning ODE is that geometric and qualitative solution.

The learning method of the geometric and qualitative solution has a potential approach to get a solution of the ODE. Some articles discussed the geometric and qualitative solution learning for several learning concepts such as graphical solutions, slope fields, and interpretation of behavior solutions (Keene, 2007; Rasmussen & Blumenfeld, 2007; Rasmussen & Keene, 2019). Research related to teaching and learning differential equations has evolved from examining elements associated with teaching in traditional classrooms to adopting a qualitative and numerical approach, modeling and the use of technology, active learning methods, and the Importance of students' involvement in their learning is emphasized. It is also characterized by modeling research and interdisciplinary due to the nature of differential equations used to describe various phenomena.

2. The topic of Learning ODE

Table 4. The topic of Learning ODE

No	Topic of learning	Paper Code	Number of records
1.	First order of ODE	A1, A2, B1, B2, B3 C1, D1, D2, D4, D6, D7, D9, D10, D12, E5	15
2.	Euler Method	D8	1
3	Application to the problem (rate of change, population model, logistic generalized, and spring-mass)	D1, D2, D3, D5, D10, E2	6
4	Second-order of ODE	B3, D1, D10	3
5	System of ODE	D1, E2	2

The classification of ODE is a separable differential equation, homogenous differential equation, exact differential equation, linear differential equation, and Bernoulli differential equation. More research discussed the topic in the first order of ODE. However, the references reviewed do not discuss other relevant concepts, techniques, and classical results related to the study of the qualitative behavior of solutions and some properties of solutions derived from the qualitative behavior. To name just a few concepts, the teaching of linear and nonlinear equations is implicitly covered in some articles.

The teaching of autonomous and non-autonomous systems and the concepts surrounding stability in nonlinear systems remain open topics for research. The teaching of advanced techniques and results for studying nonlinear systems remains open. We found no research teaching periodicity of solutions, equilibrium point analysis for nonlinear systems, and asymptotic behavior of solutions. Furthermore, the existence of positive solutions is not yet dealt with in the modeling of biological and physical problems. In short, the didactic implementation of some topics of the theory of ordinary differential equations is still open.

CONCLUSIONS

The systematic literature review found four methods of learning ODE (active learning, mathematical modeling, information, and technology communication, and geometric and qualitative solutions). Moreover, we also concluded that the several topics of ODE in the academic literature are 1) first order of ODE, 2) Euler method, 3) application to the problem (rate of change, population model, logistic generalized, and spring-mass), 4) second-order of ODE, and 5) system of ODE. The result of this study can provide a summary of existing literature and identify the weakness or gap to be investigated further in the next research related to the topic ODE.

AUTHOR CONTRIBUTIONS STATEMENT

Z devised the project and the main conceptual ideas. RIIP supervised the project. All author contributed to the interpretation of the results. EK worked out almost technical details and wrote the final version of the manuscript.

REFERENCES

Crouch, C. H., & Mazur, E. (2001). Peer instruction: Ten years of experience and results. *American Journal of Physics*, *69*(9), 970–977.

Czocher, J. A. (2017). How can emphasizing mathematical modeling principles benefit students in a traditionally taught differential equations course?. *The Journal of Mathematical Behavior*, *45*, 78–94.

de Almeida, L. M. W. (2018). Considerations on the use of mathematics in modeling activities. *ZDM*, *50*(1–2), 19–30.

Goodchild, S., Apkarian, N., Rasmussen, C., & Katz, B. (2021). Critical stance within a community of inquiry in an advanced mathematics course for pre-service teachers. *Journal of Mathematics Teacher Education*, *24*(3), 231–252.

Habre, S. (2000). Exploring students' strategies to solve ordinary differential equations in a reformed setting. *The Journal of Mathematical Behavior*, *18*(4), 455–472.

Hoyles, C. (2018). Transforming the mathematical practices of learners and teachers through digital technology. *Research in Mathematics Education*, *20*(3), 209–228.

Ju, M.-K., & Kwon, O. N. (2007). Ways of talking and ways of positioning: Students' beliefs in an inquiry-oriented differential equations class. *The Journal of Mathematical Behavior*, *26*(3), 267–280.

Kashefi, H., Ismail, Z., Yusof, Y. M., & Rahman, R. A. (2012). Fostering mathematical thinking in the learning of multivariable calculus through computer-based tools. *Procedia - Social and Behavioral Sciences*, *46*, 5534–5540.

Keene, K. A. (2007). A characterization of dynamic reasoning: Reasoning with time as parameter. *The Journal of Mathematical Behavior*, *26*(3), 230–246.

Khan, K. S., Kunz, R., Kleijnen, J., & Antes, G. (2003). Five steps to conducting a systematic review. *Journal of the royal society of medicine*, *96*(3), 118-121.

Quinn, D., & Aarão, J. (2020). Blended learning in first year engineering mathematics. *ZDM*, *52*(5), 927–941.

Rasmussen, C., & Blumenfeld, H. (2007). Reinventing solutions to systems of linear differential equations: A case of emergent models involving analytic expressions. *The Journal of Mathematical Behavior*, *26*(3), 195–210.

Rasmussen, C., & Keene, K. (2019). Knowing solutions to differential equations with rate of change as a function: Waypoints in the journey. *The Journal of Mathematical Behavior*, *56*, 100695.

Rasmussen, C., & Kwon, O. N. (2007). An inquiry-oriented approach to undergraduate mathematics. *The Journal of Mathematical Behavior*, *26*(3), 189–194.

Rasmussen, C. L. (2001). New directions in differential equations: A framework for interpreting students' understandings and difficulties. *The Journal of Mathematical Behavior*, *20*(1), 55-87.

Soares, D. da S., & Borba, M. C. (2014). The role of software modellus in a teaching approach based on model analysis. *ZDM*, *46*(4), 575–587.

Stephan, M., & Rasmussen, C. (2002). Classroom mathematical practices in differential equations. *The Journal of Mathematical Behavior*, *21*(4), 459–490.

Xiao, Y., & Watson, M. (2019). Guidance on conducting a systematic literature review. *Journal of Planning Education and Research*, *39*(1), 93–112.

Zbiek, R. M., & Conner, A. (2006). Beyond motivation: Exploring mathematical modeling as a context for deepening students' understandings of curricular mathematics. *Educational Studies in Mathematics*, *63*(1), 89–112.